DESY 06-230

Neutrino Dark Energy – Revisiting the Stability Issue

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Abstract. A coupling between a light scalar field and neutrinos has been widely discussed as a mechanism for linking (time varying) neutrino masses and the present energy density and equation of state of dark energy. However, it has been pointed out that the viability of this scenario in the non-relativistic neutrino regime is threatened by the strong growth of hydrodynamic perturbations associated with a negative adiabatic sound speed squared. In this paper we revisit the stability issue in the framework of linear perturbation theory in a model independent way. The criterion for the stability of a model is translated into a constraint on the scalar-neutrino coupling, which depends on the ratio of the energy densities in neutrinos and cold dark matter. We illustrate our results by providing meaningful examples both for stable and unstable models.

PACS numbers: 13.15.+g, 64.30.+t, 64.70.Fx, 98.80.Cq

1. Introduction

Precision observations of the cosmic microwave background [1–3], the large scale structure of galaxies [4], and distant type Ia supernovae [5–8] have led to a new standard model of cosmology in which the energy density is dominated by dark energy with negative pressure, leading to an accelerated expansion of the universe.

The simplest possible explanation for dark energy is the cosmological constant which has $P = w\rho$ with $w = -1$ at all times. However, since the cosmological constant has a magnitude completely different from theoretical expectations one is naturally led to consider other explanations for the dark energy. A light scalar field rolling in a very flat potential would for instance be a candidate better motivated from high energy physics [9–11]. In the limit of a completely flat potential it would have $w = -1$. Such models are generically known as quintessence models [12–17]. The scalar field is usually assumed to be minimally coupled to matter and to curvature, but very interesting effects can occur if this assumption is relaxed (see for instance [18–24]). In general such models alleviate the required fine tuning in order to achieve $\Omega_X \sim \Omega_m$, where Ω_X and Ω_m are the dark energy and matter densities at present. Also by properly choosing the quintessence potential it is possible to achieve tracking behaviour of the scalar field so that one also avoids the extreme fine tuning of the initial conditions for the field.

Many other possibilities have been considered, like k-essence, which is essentially a scalar field with a non-standard kinetic term [25–27]. It is also possible, although not without problems, to construct models which have $w < -1$, the so-called phantom dark energy models [28–30]. Finally, there are even more exotic models where the cosmological acceleration is not provided by dark energy, but rather by a modification of the Friedmann equation due to modifications of gravity on large scales [31, 32], or even due to higher order curvature terms in the gravity Lagrangian [33–35].

A very interesting proposal is the so-called mass varying neutrino (MaVaN) model [36–38] in which a light scalar field couples to neutrinos. Due to the coupling, the mass of the scalar field does not have to be as small as the Hubble scale but can be much larger, while the model still accomplishes late-time acceleration. This scenario also holds the interesting possibility of circumventing the well-known cosmological bound on the neutrino mass [3, 4, 40–51]. The scenario is a variant of the chameleon cosmology model [52–54] in which a light scalar field couples democratically to all non-relativistic matter.

The idea in the MaVaN model is to write down an effective potential for the scalar field which as a result of the coupling contains a term related to the neutrino energy density. If the pure scalar field potential is tuned appropriately the effective potential including the neutrino contribution will have a minimum with a steep second derivative for some finite scalar field VEV. The scalar field is therefore locked in the minimum and when the minimum evolves due to changing neutrino energy density the field tracks this evolution adiabatically. This naturally leads to a dynamical effective equation of state for the combined scalar - neutrino fluid close to $w = -1$ today, and to a neutrino mass which is related to the combined neutrino-scalar field fluid's energy density $\rho_{\rm DE}$. Since $\rho_{\rm DE}$ decreases with time, also the neutrino mass varies in this kind of scenario, where its

present value is explained in terms of $\rho_{\mathrm{DE}}^{1/4}(a = 1)$. Possible tests for the MaVaN scenario can be found in Ref. [55–62].

MaVaN models, however, suffer from the problem that for some choices of scalar-neutrino couplings and scalar field potentials the combined fluid is subject to an instability once the neutrinos become non–relativistic. Effectively the scalar field mediates an attractive force between neutrinos which can possibly lead to the formation of neutrino nuggets [63]. This in turn would make the combined fluid behave like cold dark matter and thus render it non-viable as a candidate for dark energy.

In perturbation theory the formation of these nuggets can be seen as a consequence of an imaginary speed of sound for the combined fluid, signaling fast growth of instabilities. However, an imaginary speed of sound cannot be generally used as a sufficient criterion for the instabilities, as the drag provided by cold dark matter may postpone those instabilities.

The instability can possibly occur in these models because the effective mass associated with the scalar field is much larger than H. Accordingly, on sub-Horizon scales larger than the effective Compton wavelength of the scalar field $m_\phi^{-1} < a/k < H^{-1}$ the perturbations are adiabatic.

This is a consequence of the steepness of the effective potential and can be remedied by making the potential sufficiently flat. In this case the evolution of the field is highly non-adiabatic [64, 65]. However, this model has the disadvantage that the neutrino mass is no longer related naturally to the dark energy density and equation of state.

In this paper we study various choices of scalar-neutrino couplings and scalar field potentials with the aim of identifying the conditions for the instability to occur. In the next section we review the formalism needed to study mass varying neutrinos and in section 3 we derive the equation of motion of the neutrino perturbations. Section 4 contains our results for various couplings and potentials, and finally section 5 contains a discussion and conclusion.

2. Formalism

The idea in the so-called Mass Varying Neutrino (MaVaN) scenario [36–38] is to introduce a coupling between (relic) neutrinos and a light scalar field and to identify this coupled fluid with dark energy. As a direct consequence of this new interaction, the neutrino mass m_ν is generated from the vacuum expectation value (VEV) of the scalar field and becomes linked to its dynamics. Thus the pressure $P_\nu(m_\nu(\phi), a)$ and energy density $\rho_\nu(m_\nu(\phi), a)$ of the uniform neutrino background contribute to the effective potential $V(\phi, a)$ of the scalar field. The effective potential is defined by

$$V(\phi) = V_\phi(\phi) + (\rho_\nu - 3P_\nu) \tag{1}$$

where $V_\phi(\phi)$ denotes the fundamental scalar potential and a is the scale factor. Throughout the paper we assume a flat Friedman-Robertson-Walker cosmology and use the convention $a_0 = 1$, where we take the subscript 0 to denote present day values.

Assuming the neutrino distribution to be Fermi-Dirac and neglecting the chemical potential, the energy density and pressure of the neutrinos can be expressed in the following

form [39]

$$\begin{aligned}\rho_\nu(a,\phi) &= \frac{T_\nu^4(a)}{\pi^2}\int_0^\infty \frac{dy\, y^2\sqrt{y^2+\frac{m_\nu^2(\phi)}{T_\nu^2(a)}}}{e^y+1},\\ P_\nu(a,\phi) &= \frac{T_\nu^4(a)}{3\pi^2}\int_0^\infty \frac{dy\, y^4}{\sqrt{y^2+\frac{m_\nu^2(\phi)}{T_\nu^2(a)}}(e^y+1)},\end{aligned} \tag{2}$$

where $T_\nu = T_{\nu_0}/a$ is the neutrino temperature and y corresponds to the ratio of the neutrino momentum and neutrino temperature, $y = p_\nu/T_\nu$.

The energy density and pressure of the scalar field are given by the usual expressions,

$$\begin{aligned}\rho_\phi(a) &= \frac{1}{2a^2}\dot{\phi}^2 + V_\phi(\phi),\\ P_\phi(a) &= \frac{1}{2a^2}\dot{\phi}^2 - V_\phi(\phi).\end{aligned} \tag{3}$$

Defining $w = P_{\rm DE}/\rho_{\rm DE}$ to be the equation of state of the coupled dark energy fluid, where $P_{\rm DE} = P_\nu + P_\phi$ denotes its pressure and $\rho_{\rm DE} = \rho_\nu + \rho_\phi$ its energy density, and the requirement of energy conservation gives,

$$\dot{\rho}_{DE} + 3H\rho_{DE}(1+w) = 0. \tag{4}$$

Here $H \equiv \frac{\dot{a}}{a}$ and we use dots to refer to the derivative with respect to conformal time. Taking Eq. (4) into account, one arrives at a modified Klein-Gordon equation describing the evolution of ϕ,

$$\ddot{\phi} + 2H\dot{\phi} + a^2 V_\phi' = -a^2\beta(\rho_\nu - 3p_\nu). \tag{5}$$

Here and in the following primes denote derivatives with respect to ϕ ($' = \partial/\partial\phi$) and $\beta = \frac{d\log m_\nu}{d\phi}$ is the coupling between the scalar field and the neutrinos.

2.1. The fully adiabatic case

In the following let us consider the late time evolution of the coupled scalar-neutrino fluid in the limit $m_\nu \gg T_\nu$ where the neutrinos are non-relativistic. It is in this regime that MaVaN models can potentially become unstable for the following reason: The attractive force mediated by the scalar field (which can be much stronger than gravity) acts as a driving force for the instabilities. But as long as the neutrinos are still relativistic, the evolution of the density perturbations will be dominated by pressure which inhibits their growth, as the strength of the coupling is suppressed when $\rho_\nu = 3P_\nu$.

In the non-relativistic limit $m_\nu \gg T_\nu$, the expressions for the energy density and pressure in neutrinos in Eq. (2) reduce to

$$\begin{aligned}\rho_\nu &\simeq m_\nu n_\nu,\\ P_\nu &\simeq 0,\end{aligned} \tag{6}$$

such that Eq. (1) takes the form

$$V = \rho_\nu + V_\phi = m_\nu n_\nu + V_\phi. \tag{7}$$

Assuming the curvature scale of the potential and thus the mass of the scalar field m_ϕ to be much larger than the expansion rate of the Universe,

$$V'' = \rho_\nu \left(\beta' + \beta^2\right) + V''_\phi \equiv m_\phi^2 \gg H^2, \tag{8}$$

the adiabatic solution to the equation of motion of the scalar field in Eq. (5) applies [38]‡. As a consequence, the scalar field instantaneously tracks the minimum of its effective potential V, solution to the condition

$$V' = \rho'_\nu + V'_\phi = m'_\nu \left(\frac{\partial \rho_\nu}{\partial m_\nu} + \frac{\partial V_\phi}{\partial m_\nu}\right) = m'_\nu \left(n_\nu + \frac{\partial V_\phi}{\partial m_\nu}\right) = 0. \tag{9}$$

As the universe expands the neutrino energy density gets diluted, thus naturally giving rise to a slow evolution of $V(\phi)$. Consequently, the value of the scalar field ϕ evolves on cosmological time scales. Note that as long as m'_ν does not vanish, this implies that also the neutrino mass $m_\nu(\phi)$ is promoted to a time dependent, dynamical quantity. Its late time evolution can be determined from the last equality in Eq. (9).

In order to specify good candidate potentials $V_\phi(\phi)$ for a viable MaVaN model of dark energy, we must demand that the equation of state parameter w of the coupled scalar-neutrino fluid today roughly satisfies $w \sim -1$ as suggested by observations [66]. By noting that for constant w at late times,

$$\rho_{\rm DE} \sim V \propto a^{-3(1+w)} \tag{10}$$

and by requiring energy conservation according to Eq. (4), one arrives at [38]

$$1 + w = -\frac{1}{3}\frac{\partial \log V}{\partial \log a}. \tag{11}$$

In the non-relativistic limit $m_\nu \gg T_\nu$ this is equivalent to

$$1 + w = -\frac{a}{3V}\left(m_\nu \frac{\partial n_\nu}{\partial a} + n_\nu \frac{\partial m_\nu}{\partial a} + \frac{V'_\phi}{a'}\right) = -\frac{m_\nu V'_\phi}{m'_\nu V}, \tag{12}$$

where in the last equality it has been used that $V' = 0$ according to Eq. (9). To allow for an equation of state close to $w \sim -1$ today one can conclude that either the scalar potential V_ϕ has to be fairly flat or the dependence of the neutrino mass on the scalar field has to be very steep.

2.2. The general case

As it will turn out later, the influence of the cosmic expansion in combination with the gravitational drag exerted by CDM on the neutrinos can have an effect on the stability of a MaVaN model. However, to begin we will neglect any growth-slowing effects on the perturbations and proceed with a more general analysis of this case. Under these circumstances, the dynamics of the perturbations are solely determined by the sound speed squared which for a general fluid component i takes the following form,

$$c_{si}^2 = \frac{\delta P_i}{\delta \rho_i}, \tag{13}$$

‡ In this case for $|\phi| < M_{\rm pl} \simeq 3 \times 10^{18}$ GeV the effects of the kinetic energy terms can be safely ignored [38].

where P_i and ρ_i denote the fluid's pressure and energy density, respectively. The sound speed c_{si}^2 can be expressed in terms of the sound speed c_{ai}^2 arising from purely adiabatic perturbations as well as an additional entropy perturbation Γ_i and the density contrast $\delta_i = \delta\rho_i/\rho_i$ in the given frame [67, 68],

$$w_i\Gamma_i = (c_{si}^2 - c_{ai}^2)\,\delta_i, \tag{14}$$

$$= \frac{\dot{P}_i}{\rho_i}\left(\frac{\delta P_i}{\dot{P}_i} - \frac{\delta\rho_i}{\dot{\rho}_i}\right). \tag{15}$$

Here w_i denotes the equation of state parameter and Γ_i is a measure for the relative displacement between hypersurfaces of uniform pressure and uniform energy density. For most dark energy candidates (like quintessence or k-essence) dissipative processes evoke entropy perturbations and thus $\Gamma_i \neq 0$.

However, in MaVaN models the effective mass of the scalar field $m_\phi \gg H$ sets the scale, m_ϕ^{-1}, where these processes and the associated gradient terms become unimportant [63, 69], to be much smaller than the Hubble radius (in contrast to a quintessence field with finely-tuned mass $\lesssim H$ and long range $\gtrsim H^{-1}$). As a consequence, on sub-Hubble scales $H^{-1} > \frac{a}{k} > m_\phi^{-1}$ all dynamical properties of (non-relativistic) MaVaNs are set by the local neutrino energy density [63]. In particular, for small deviations away from the minimum of its effective potential, the scalar field re-adjusts to its new minimum on time scales $\sim m_\phi^{-1}$ small compared to the characteristic cosmological time scale H^{-1}. In this case the hydrodynamic perturbations in MaVaNs are adiabatic. This means the system of neutrinos and the scalar field can be treated as a unified fluid with pressure $P_{DE} = P_\nu + P_\phi$ and energy density $\rho_{DE} = \rho_\nu + \rho_\phi$ without intrinsic entropy, $\Gamma_{DE} = 0$ §.

If any growth-slowing effects can be neglected, the perturbations in a MaVaN model are driven by the effective sound speed squared given by

$$c_a^2 = \frac{\dot{P}_{\rm DE}}{\dot{\rho}_{DE}} = \frac{\dot{w}\rho_{DE} + w\dot{\rho}_{DE}}{\dot{\rho}_{DE}} = w - \frac{\dot{w}}{3H(1+w)}, \tag{16}$$

where Eq. (4) and Eq. (15) have been used. In the case $c_a^2 > 0$ the attractive scalar force is offset by pressure forces and the fluctuations oscillate as sound waves and can be considered as stable. However, for $c_a^2 < 0$ perturbations become unstable and tend to blow up.

3. Evolution of the Perturbations

In this section we will analyse the linear MaVaN perturbations in the synchronous gauge, which is characterised by a perturbed line element of the form

$$ds^2 = a(\tau)^2(-d\tau^2 + (\delta_{ij} + h_{ij})dx^i dx^j), \tag{17}$$

where τ denotes conformal time and h_{ij} is the metric perturbation. Here and in the following dots represent derivatives with respect to τ. Most of our other notations and

§ (see also [70] for another example of unified models)

conventions comply with those in Ma and Bertschinger [74]. Consequently, the Friedmann equation takes the form

$$3H^2 = \frac{a^2}{M_{\rm pl}^2}\left(\frac{\dot\phi^2}{2a^2} + V_\phi(\phi) + \rho_m\right), \tag{18}$$

with $M_{\rm pl} \equiv (\sqrt{8\pi G})^{-1}$ denoting the reduced Planck mass and the subscript m comprising all matter species.

Since the following perturbation equations have been widely discussed in the literature (e.g. [23, 53, 65, 76, 77] and references therein), we will simply state them here for neutrinos coupled to a scalar field.

The evolution equation for the MaVaN density contrast $\delta_\nu = \delta\rho_\nu/\rho_\nu$ is given by [65],

$$\begin{aligned}\dot\delta_\nu = {}& 3\left(H + \beta\dot\phi\right)\left(w_\nu - \frac{\delta p_\nu}{\delta\rho_\nu}\right)\delta_\nu - (1+w_\nu)\left(\theta_\nu + \frac{\dot h}{2}\right)\\ &+ \beta\,(1-3w_\nu)\,\delta\dot\phi + \beta'\dot\phi\delta\phi\,(1-3w_\nu)\,,\end{aligned} \tag{19}$$

where $\beta = \frac{d\log m_\nu}{d\phi}$.

Furthermore, the trace of the metric perturbation, $h \equiv \delta^{ij}h_{ij}$, according to the linearised Einstein equations satisfies,

$$\ddot h + H\dot h = \frac{a^2}{M_{\rm pl}^2}[\delta T^0_0 - \delta T^i_i], \text{ where} \tag{20}$$

$$\delta T^0_0 = -\frac{1}{a^2}\dot\phi\delta\dot\phi - V'_\phi(\phi)\delta\phi - \sum_m \rho_m\delta_m, \tag{21}$$

$$\delta T^i_i = \frac{3}{a^2}\dot\phi\delta\dot\phi - 3V'_\phi(\phi)\delta\phi + \sum_r \rho_r\delta_r + 3c_b^2\rho_b\delta_b + 3c_\nu^2\rho_\nu\delta_\nu. \tag{22}$$

Here δT^μ_ν denotes the perturbed stress energy tensor and the subscripts m and r collect neutrinos, radiation, CDM and baryons (with sound speed c_b) as well as (relativistic) neutrinos and radiation, respectively.

The evolution equation for the neutrino velocity perturbation $\theta_\nu \equiv ik_iv^i_\nu$ with $v^i_\nu \equiv dx^i/d\tau$ reads [65],

$$\begin{aligned}\dot\theta_\nu = {}& -H(1-3w_\nu)\theta_\nu - \frac{\dot w_\nu}{1+w_\nu}\theta_\nu + \frac{\frac{\delta p_\nu}{\delta\rho_\nu}}{1+w_\nu}k^2\delta_\nu\\ &+ \beta\frac{1-3w_\nu}{1+w_\nu}k^2\delta\phi - \beta(1-3w_\nu)\,\dot\phi\,\theta_\nu - k^2\sigma_\nu,\end{aligned} \tag{23}$$

where σ_ν denotes the neutrino shear as defined in [74].

Finally, the perturbed Klein-Gordon equation for the coupled scalar field is given by [65]

$$\begin{aligned}&\delta\ddot\phi + 2H\delta\dot\phi + \left[k^2 + a^2\left\{V''_\phi + \beta'(\rho_\nu - 3P_\nu)\right\}\right]\delta\phi + \frac{1}{2}\dot h\dot\phi =\\ &- a^2\beta\delta_\nu\rho_\nu(1 - 3\frac{\delta p_\nu}{\delta\rho_\nu}).\end{aligned} \tag{24}$$

We note that instead of proceeding via the fluid equations, Eqs. (19) and (23), the evolution of the neutrino density contrast can be calculated from the Boltzmann equation

[74]. We have verified analytically and numerically that the two methods yield identical results provided that the scalar-neutrino coupling is appropriately taken account of in the Boltzmann hierarchy [75].

As discussed in sec. 2 MaVaNs models can only possibly become unstable on sub-Hubble scales $m_\phi^{-1} < a/k < H^{-1}$ in the non-relativistic regime of the neutrinos, where the perturbations evolve adiabatically. For our numerical results in the next section we solve the coupled Eqs. (19-24) in the (quasi-)adiabatic regime by neglecting the neutrino shear σ_ν. This approximation is justified, since the scalar-neutrino coupling becomes important in this regime and m_ν is much larger than the mean momentum of the neutrino distribution.

For the purpose of gaining further analytical insight into the evolution of the neutrino density contrast, it is instructive to apply additional approximations to Eqs. (19-24) to be justified in the following.

Since the minimum of the effective potential tracked by the scalar field evolves only slowly due to changes in the neutrino energy density, we can safely ignore terms proportional to $\dot{\phi}$. Moreover, in the non-relativistic regime of the neutrinos on scales $m_\phi^{-1} < a/k < H^{-1}$, as a consequence of $P_\nu \sim 0$ it follows that $\sigma_\nu \sim 0$ and $w_\nu \sim 0$ as well as $\rho_r \sim c_b^2 \sim 0$. In addition, in the following we substitute $\delta\phi$ by its average value corresponding to the forcing term on the right hand side of Eq. (24) in the above limits,

$$\delta\bar{\phi} = -\frac{\beta \rho_\nu \delta_\nu}{(V_\phi'' + \rho_\nu \beta') + \frac{k^2}{a^2}}, \tag{25}$$

which solves the perturbed Klein-Gordon equation reasonably well on all scales [23, 76]. Finally, by combining the derivative of Eq. (19) with Eq. (20) – Eq. (23) and Eq. (25) in the non-relativistic limit, we arrive at the equation of motion for the neutrino density contrast valid at late times on length scales $m_\phi^{-1} < a/k < H^{-1}$,

$$\ddot{\delta}_\nu + H\dot{\delta}_\nu + \left(\frac{\delta p_\nu}{\delta \rho_\nu} k^2 - \frac{3}{2} H^2 \Omega_\nu \frac{G_{\rm eff}}{G}\right) \delta_\nu = \frac{3}{2} H^2 \Big[\ \Omega_{\rm CDM} \delta_{\rm CDM} + \Omega_b \delta_b\ \Big] \tag{26}$$

where

$$G_{\rm eff} = G \left(1 + \frac{2\beta^2 M_{\rm pl}^2}{1 + \frac{a^2}{k^2}\{V_\phi'' + \rho_\nu \beta'\}}\right) \text{ and} \tag{27}$$

$$\Omega_i = \frac{a^2 \rho_i}{3H^2 M_{\rm pl}^2}. \tag{28}$$

Since neutrinos not only interact through gravity, but also through the force mediated by the scalar field, they feel an effective Newton's constant $G_{\rm eff}$ as defined in Eq. (27). The force depends upon the MaVaN model specific functions β and V_ϕ and takes values between G and $G(1 + 2\beta^2 M_{\rm pl}^2)$ on very large and small length scales, respectively. The scale dependence of $G_{\rm eff}$ is due to the finite range of the scalar field $(V_\phi'' + \rho_\nu\beta')^{-\frac{1}{2}}$, which according to Eq. (8) is equal to $(m_\phi^2 - \beta^2\rho_\nu)^{-\frac{1}{2}}$. For moderate coupling strength it is essentially given by the inverse scalar field mass, whereas for $\beta \gg 1/M_{\rm pl}$ it can take larger values. Accordingly, in a MaVaN model both the scalar potential V_ϕ and the coupling

β influence the range of the scalar field force felt by neutrinos, whereas its strength is determined by the coupling β.

The evolution of perturbations in cold dark matter (CDM) coupled to a light scalar field in coupled quintessence [23] and chameleon cosmologies [53] is governed by an equation similar to Eq. (26). However, we would like to point out that for the same coupling functions the dynamics of the perturbations in neutrinos can be quite different from those in coupled CDM. This is a result of the fact that $\Omega_\nu \ll (\Omega_{\rm CDM} + \Omega_{\rm b})$. Whereas $\Omega_{\rm CDM} \sim 0.2$ and $\Omega_{\rm b} \sim 0.05$ [4] at present, Ω_ν depends on the so far not known absolute neutrino mass scale realised in nature. Taking as a lower bound the mass splitting deduced from atmospheric neutrino flavour oscillation experiments and the upper bound derived from the Mainz tritium beta-decay experiments [81], we get $10^{-4} \lesssim \Omega_\nu \lesssim 0.15$ today ‖. It is important to note that since in the standard MaVaN scenario the neutrino mass is an increasing function of time, at earlier times the ratio $\Omega_\nu/(\Omega_{\rm CDM}+\Omega_{\rm b})$ was even smaller than today. In general it follows that the smaller this ratio is, the larger the relative influence of the forcing term on the RHS of Eq. (26) becomes. The forcing term describes the effect of the perturbations of other cosmic components on the dynamics of the neutrino density contrast and competes with the scalar field dependent term $\propto \frac{G_{\rm eff}}{G}\Omega_\nu\delta_\nu$ on the LHS. Correspondingly, apart from the scalar field mediated force the neutrinos feel the gravitational drag exerted by the potential wells formed by CDM. Consequently, as long as the coupling function β does not considerably enhance the influence of the term $\propto \frac{G_{\rm eff}}{G}\Omega_\nu\delta_\nu$, the non-relativistic neutrinos will follow CDM (like baryons) just as in the Standard Model.

In the following we classify the behaviour of the neutrino density contrast in models of neutrino dark energy subject to all relevant kinds of coupling functions β. We emphasize that this classification is completely model independent. In the small-scale limit we distinguish the following three cases:

Small-scale limit

a) For $\Omega_\nu(1 + 2\beta^2 M_{\rm pl}^2) < \Omega_{\rm CDM}$ until the present time, the neutrino density contrast is stabilised by the CDM source term which dominates its dynamics. In this case the influence of the scalar field on the perturbations is subdominant and the density contrast in MaVaNs grows moderately just like gravitational instabilities in uncoupled neutrinos.

b) For $\beta \sim$ const. and much larger than all other parameters at late times, $G_{\rm eff} \gg G$, the damping term $H\dot{\delta}_\nu$ in Eq. (26) as well as the the terms proportional to $\delta_{\rm CDM}$ and δ_b can be neglected, leading to exponentially growing solutions.

c) For $\beta \neq$ const. and growing faster than all other parameters at late times, $G_{\rm eff} \gg G$, δ_ν is growing faster than exponentially¶.

‖ Note that if the upper limit from the Mainz experiment is saturated the requirement $\Omega_\nu \ll \Omega_m$ is formally not satisfied. However, this case should be viewed as very extreme and is most likely excluded based on structure formation arguments

¶ In the limit $\beta(\tau) \to \infty$ for $\tau \to \infty$, Eq. 26 takes the form $\ddot{\delta}_\nu - 3H^2\Omega_\nu \frac{\beta^2(\tau)M_{\rm pl}^2}{1+a^2(V''_\phi+\rho_\nu\beta')/k^2}\delta_\nu = 0$, and it can be shown that $|\frac{\dot{\delta}_\nu}{\delta_\nu}| \to \infty$ for $\tau \to \infty$ [79]. Since this ratio is constant and thus not large enough for an exponentially growing δ_ν, the solution is required to grow faster than exponentially.

In contrast, on scales $(V''_\phi + \rho_\nu \beta')^{-1/2} \ll a/k < H^{-1}$ much larger than the range of the ϕ-mediated force,

Large-scale limit

d) For $\beta \sim$ const. and of moderate strength, $G_{\rm eff} \sim G$ and the perturbations behave effectively like perturbations for uncoupled fluids in General Relativity.

e) For β growing faster than all other quantities at late times, $G_{\rm eff} \gg G$, instabilities develop on all sub-Hubble scales $a/k > (V''_\phi + \rho_\nu \beta')^{-1/2}$ according to *c*). However, on large length scales their growth rate is suppressed due to the corresponding small wave number k.

3.1. Potentials and Couplings

In the following, we consider two combinations of scalar potentials $V_\phi(\phi)$ and of scalar-neutrino couplings β which define our MaVaN models. The potentials are chosen to accomplish the required cosmic late-time acceleration and for the couplings we take meaningful limiting cases.

Our main point is to present a proof of concept of the stability conditions stated above, which is valid for a general adiabatic MaVaN model. We note that a certain degree of fine-tuning is exerted. It is mainly due to the fact that CMBFAST and CAMB only operate in the linear regime $H \sim 10^{-4}\text{Mpc}^{-1} < k < 0.1\text{Mpc}^{-1}$ and correspondingly, only in this regime can we analytically track the evolution of perturbation by the help of linear theory. Since the Compton wavelength of the scalar field $\sim m_\phi^{-1}$ sets the length scale of interest where possible instabilities can grow fastest, $a/k \gtrsim m_\phi^{-1}$, (cf. the discussion in sec. 2), this implies the scalar field mass has to be tuned accordingly, while at the same time the correct cosmology has to be accomplished.

Firstly, we consider a MaVaN model suggested by [38] which we will refer to as the *log-linear model*. The scalar field has a Coleman-Weinberg type [78] logarithmic potential,

$$V_\phi(\phi) = V_0 \log(1 + \kappa\phi), \tag{29}$$

where the constants V_0 and κ are chosen appropriately to yield $\Omega_{\rm DE} \sim 0.7$ and $m_\phi \gg H$ today. The choice of V_ϕ determines the evolution of ϕ according to Eq. (7) as plotted in fig. 1. Apparently, the neutrino background has a stabilising effect on ϕ. It drives the scalar field to larger values and stops it from rolling down its potential V_ϕ. This competition of the two terms in Eq. (7) results in a minimum at an intermediate value of ϕ (cf. Eq. 9), which slowly evolves due to changes in the neutrino energy density. As the universe expands and ρ_ν dilutes, both the minimum and the scalar field are driven to smaller values towards zero.

Let us now turn to the neutrino mass and its evolution. The dependence of m_ν on the scalar field is given by,

$$m_\nu(\phi) = m_0 \frac{\phi_0}{\phi}. \tag{30}$$

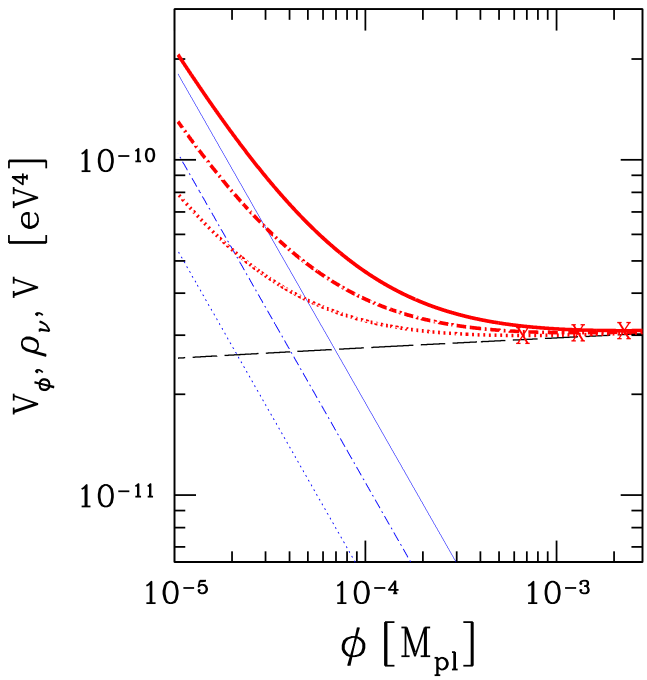

Figure 1. The effective potential V (thick lines), composed of the scalar potential V_ϕ (dashed) and the neutrino energy density ρ_ν, plotted for three different redshifts, $z = 5$ (solid), $z = 4$ (dashed-dotted), $z = 3$ (dotted). The VEV of ϕ tracks the minimum of V (marked by X) and evolves to smaller values for decreasing redshift. We have used $\kappa = 1 \times 10^{20} M_{\rm pl}^{-1}$ and $V_0 = 8.1 \times 10^{-13}\mathrm{eV}^4$.

Such a dependence naturally emerges in the framework of the seesaw mechanism. In this case the light neutrino mass m_ν arises from integrating out a heavier sterile state, whose mass varies linearly with the value of the scalar field (as e.g. in Ref. [38, 63, 72]).

According to Eq. (30) this model is characterised by a field dependent coupling,

$$\beta(\phi) = \frac{1}{m}\frac{\partial m}{\partial \phi} = -\frac{1}{\phi}, \tag{31}$$

which corresponds to a time evolution as plotted in fig. 2.

Since the value of ϕ decreases with time (cf. fig. 1) this means the rate of energy transfer between the scalar field and the neutrinos and also the attraction felt between neutrinos increases with time. Consequently, both the neutrino mass m_ν in Eq. (30) and according to Eq. (2) also the neutrino energy density blow up when ϕ approaches zero. Thus, from these qualitative considerations it can already be expected that the model will run into stability problems in the non-relativistic neutrino regime.

Secondly, we consider an inverse power-law model, which we will refer to as the *power-model*.

$$V_\phi = \frac{M^{n+4}}{\phi^n}, \tag{32}$$

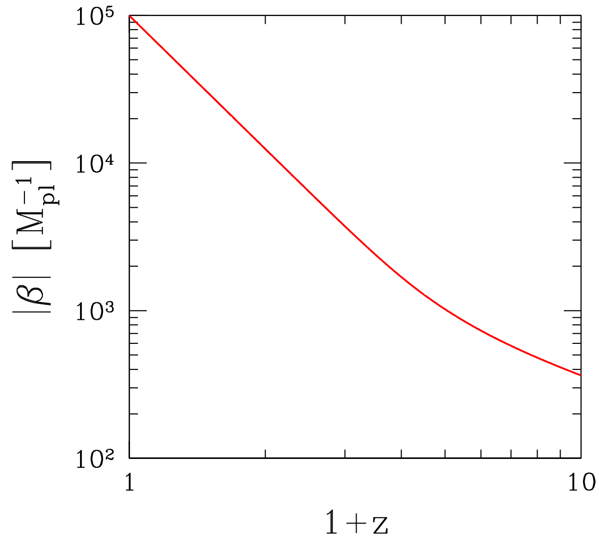

Figure 2. The evolution of the effective coupling, β (given by Eq. (31)), as a function of redshift for the potential Eq. (29). We have used $\kappa = 1 \times 10^{20} M_{\rm pl}^{-1}$ and $V_0 = 8.1 \times 10^{-13}\mathrm{eV}^4$.

Note that this is similar to a model proposed in the context of chameleon cosmologies [52, 53, 80]. However, there are some notable differences: Our potential does not reduce to a cosmological constant in the asymptotic future and $V(\phi) \to \infty$ for $\phi \to 0$. The last point is, however, not problematic from a cosmology point of view since for realistic value of the power-law exponent n it is always true that $\Omega_\phi \to 0$ for $t \to 0$.

The mass parameter M is fixed by the requirements $\Omega_{DE} \sim 0.7$ and $m_\phi \gg H$. In fig. 3 the evolution of ϕ is plotted according to Eq. (1). In contrast to the first model, the expectation value of ϕ is increasing with time.

In this model the dependence of the neutrino mass on the scalar field is taken to be,

$$m_\nu = m_0 e^{\sigma\phi^2}, \tag{33}$$

where σ is a constant. The power-law model is characterised by a field-dependent coupling,

$$\beta = \frac{1}{m_\nu}\frac{\partial m_\nu}{\partial \phi} = 2\sigma\phi, \tag{34}$$

which corresponds to a time evolution as plotted in fig. 4.

Since according to fig. 3 the value of ϕ is increasing until the present time, the mass and consequently also the coupling is growing with time - cf. fig. 4. The growth of the mass depends on the choice of the parameter σ and can be quite moderate compared to the log-linear model. This can make the model more stable.

4. Results

In this section we present the numerical results of our stability analysis for the two MaVaN models of the last section. They are obtained from modifying the CMBFAST code [82]

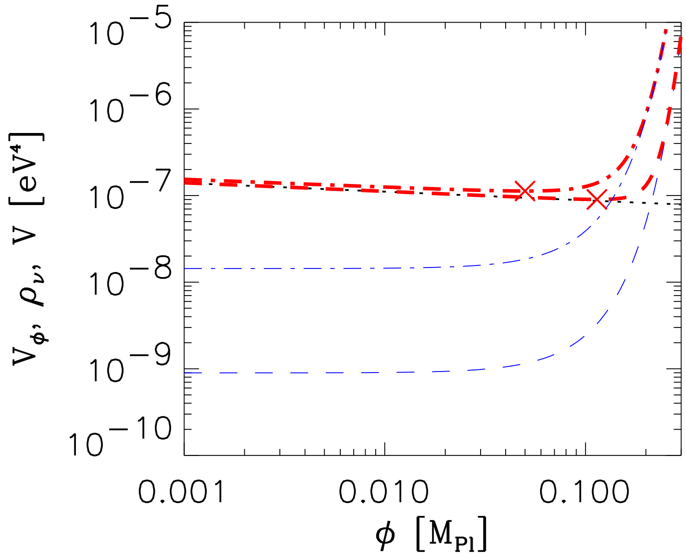

Figure 3. The effective potential V (thick lines), composed of the scalar potential V_ϕ (dotted) and the neutrino energy density ρ_ν, plotted for two different redshifts, $z = 1$ (dot-dashed), $z = 0$ (dashed). The VEV of ϕ tracks the minimum of V (marked by X) and evolves to larger values for decreasing redshift. We have used $n = 0.01$ and $M = 0.011$ eV.

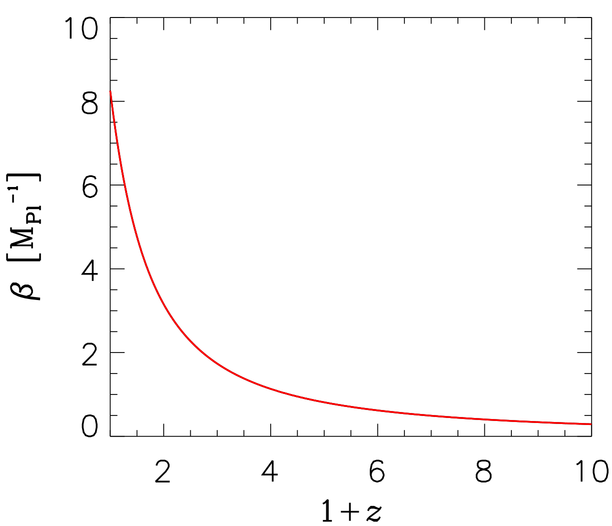

Figure 4. The evolution of the effective coupling, β (given by Eq. (34)), as a function of redshift for the potential Eq. (32). We have used $\sigma = 100 M_{\rm pl}^{-2}$.

to include a light scalar field coupled to neutrinos and were checked by altering the CAMB code [83] accordingly. We assume a neutrino energy density of $\Omega_\nu \sim 0.02$, which corresponds roughly to the current conservative upper limit on the sum of neutrino masses from CMB and LSS data [3, 4, 40] $^+$, where we take the present day normalised Hubble expansion rate to be $h = 0.7$. Ω_ν corresponds to the energy density of three neutrino species

$^+$ Note those constraints were obtained assuming non-interacting neutrino models. Hence this assumption could be relaxed.

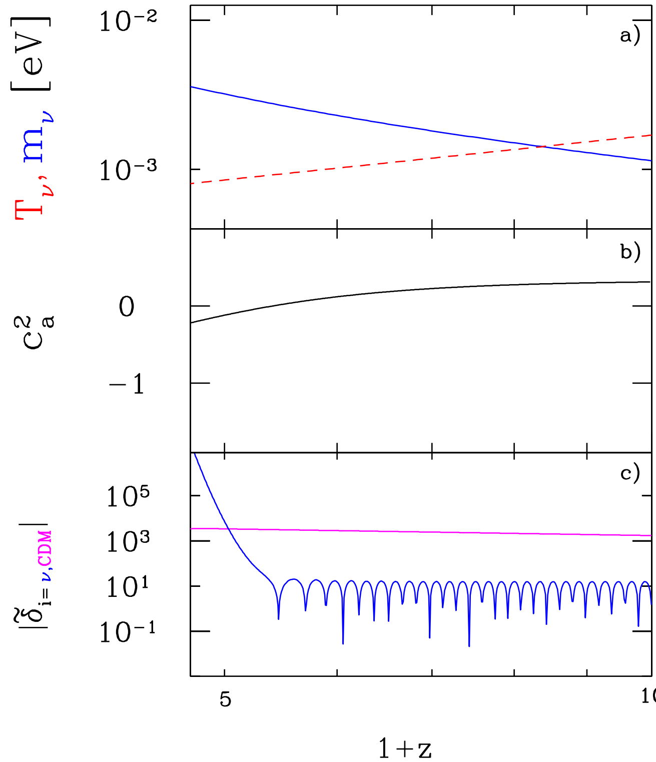

Figure 5. *a*) Neutrino mass m_ν (solid) and temperature T_ν (dotted) as a function of redshift. *b*) Total dark energy sound speed squared c_a^2 as a function of redshift. *c*) Density contrast in neutrinos (oscillating) δ_ν and density contrast in CDM $\delta_{\rm CDM}$ as a function of redshift on a scale $k = 0.1\,{\rm Mpc}^{-1}$. We have used $\kappa = 1 \times 10^{20} M_{\rm pl}^{-1}$ and $V_0 = 8.1 \times 10^{-13}{\rm eV}^4$.

with degenerate mass $m_{\nu_i}(z = 0) \sim 0.312\,{\rm eV} \gg T_{\nu_0}$, which are highly non-relativistic today. We note that this particular choice of neutrino mass has no qualitative impact on our results.

4.1. Log-linear Model

The log-linear model is defined by Eq. (29) and Eq. (30). By fine-tuning the parameter V_0 for a fixed value of $\kappa = 10^{20} M_{\rm pl}^{-1}$ in Eq. (29), standard cosmology with $\Omega_{\rm DE} = 0.7$, $\Omega_{\rm CDM} = 0.25$, and $\Omega_{\rm b} = 0.05$ at present can be accomplished, where $\Omega_{\rm DE} = \Omega_\nu + \Omega_\phi$.

The mass of ϕ at present determined from Eq. (8) is $m_\phi = 5.74\ {\rm Mpc}^{-1} \gg H$. Consequently, the Compton wavelength of the scalar field, m_ϕ^{-1}, sets the scales on which the perturbations in (non-relativistic) MaVaNs are adiabatic, $m_\phi^{-1} < a/k < H^{-1}$ (cf. the discussion in sec. 2.2). We produce our results on a scale $k = 0.1{\rm Mpc}^{-1} \sim m_\phi$. In fig. 5 we present our results for the evolution of the neutrino mass, the sound speed squared and the density contrast to be discussed in the following.

a) The evolution of the neutrino mass $m_\nu(z)$ and the neutrino temperature $T_\nu(z) = T_{\nu_0}(1+z)$ is plotted as a function of redshift. As long as $m_\nu(z) \ll T_\nu(z)$, the neutrinos are relativistic, whereas for $m_\nu(z) \gg T_\nu(z)$ they have turned non-relativistic. The

transition takes place at roughly $z+1 \sim 7$, i.e. when $m_\nu(z) \simeq 3T_\nu(z)$. One interesting feature is that for $z \to 0$ the neutrino mass grows as $m_\nu(z) \propto a^3$ so that $\rho_\nu \to$ Constant.

b) A plot of the total adiabatic sound speed squared of the coupled fluid c_a^2. It decreases when the neutrinos approach the non-relativistic regime $m_\nu(z) \gg T_\nu(z)$(cf. *a*)). This is due to the drop in the neutrino pressure from initially $P_\nu \sim 1/3$ to $P_\nu \sim 0$ well after the transition of regimes.

c) A plot of the density contrast in neutrinos $\delta_\nu = \delta\rho_\nu/\rho_\nu$, and cold dark matter (CDM) $\delta_{\rm CDM} = \delta\rho_{\rm CDM}/\rho_{\rm CDM}$ on a scale of $k = 0.1$ Mpc^{-1}. As long as the neutrinos are still relativistic ($m_\nu(z) \ll T_\nu(z)$ cf. *a*)), the perturbations in the strongly coupled scalar-neutrino fluid oscillate like sound waves. However, after pressure cannot offset the attractive force anymore ($m_\nu(z) > 3T_\nu(z)$), the neutrino density contrast blows up and thus grows at a much faster rate than the density contrast in CDM (the fast growth sets in after the effective sound speed squared has turned negative). This can be understood by considering the evolution of the coupling β between the scalar field and neutrinos (cf. fig. 2), since β^2 according to Eq. (26) governs the evolution of the density contrast in non-relativistic neutrinos. The choice of a large κ corresponds to $\phi \ll M_{\rm pl}$ at late times, and hence β^2 is driven to larger and larger values, while the VEV of ϕ approaches zero (cf. the discussion in the last section). Accordingly, δ_ν is subject to an effective Newton's constant $G_{\rm eff} \gg G$ (cf. the discussion in sec. 3). However, $\delta_{\rm CDM}$ behaves essentially as in General Relativity, as long as the modification to the gravitational effect on CDM caused by the scalar-field induced change in the neutrino density contrast is not prominent. Since the coupling and thus $G_{\rm eff}$ rapidly increase with time, the scalar field transfers more and more energy to the neutrinos causing m_ν to increase (cf. *a*)). Therefore, both β as well as the energy density in neutrinos increase such that the stabilising effect of the CDM becomes less and less important and finally becomes entirely negligible.
As a further consequence, the attraction between neutrinos also rises steadily, while the neutrino pressure drops and ceases to stabilise the perturbations. As demonstrated in *b*) the total sound speed squared is thus quickly driven to negative values, causing δ_ν to grow faster than exponentially (cf. also the discussion in sec. 3). As a result, we can show that the neutrino density contrast has already turned non-linear at $z+1 \sim 5$. Hence we take into account the normalisation of the CDM density contrast which gives us a rough estimate for the normalisation of δ_ν. As long as the dimensionless power spectrum $\Delta^2(k) = k^3 P(k)/(2\pi^2) \propto \delta_{\rm CDM}^2 < 1$, CDM perturbations on a scale k are linear, where $P(k)$ denotes the power spectrum of CDM. Since on the considered scale of $k = 0.1\,{\rm Mpc}^{-1}$ we have $\Delta^2(k) \sim 0.3 - 0.4$ [84] for CDM, we can infer that for neutrinos $\Delta^2(k) \propto \delta_\nu^2 > 1$, when δ_ν exceeds $\delta_{\rm CDM}$ by more than a factor of $\sqrt{2}$. This is the case at roughly $1+z \sim 5$, while afterwards linear perturbation theory breaks down. It is thus likely that neutrinos in this model are subject to the formation of non-linear structure in the neutrino energy density [63] before the present time.

Our numerical results presented in fig. 5 demonstrate that the total sound speed squared in the log-linear model is negative at late times, corresponding to a fast growth of

perturbations. Thus, inevitably, the neutrino density contrast at some point in time will go non-linear and the model becomes unstable with the possible outcome of the formation of neutrino bound states [63].

4.2. Power-law Potential

The power-law potential is defined by Eq. (32) and Eq. (33). We have chosen $n = 0.1$ and $n = 0.01$ in Eq. (32) to guarantee an adiabatic evolution of the scalar field until the present time, where $m_\phi \sim 10^{-3}\mathrm{Mpc}^{-1} \sim 10H$ today. In addition, this choice of parameters allows us to push the scales where possible adiabatic instabilities can occur, $m_\phi^{-1} \lesssim a/k < H^{-1}$, into the linear regime. Accordingly, we perform our perturbation analysis on a scale $k = m_\phi \sim 10^{-3}\,\mathrm{Mpc}^{-1}$ and illustrate our results in fig. 6 and fig. 7 to be described in the following.

a) The evolution of the neutrino mass $m_\nu(z)$ and the neutrino temperature $T_\nu(z)$ in the non-relativistic regime $m_\nu(z) \gg T_\nu(z)$ is plotted as a function of redshift. Note that the neutrinos turn non-relativistic at quite early times compared to the log-linear model.

b) The evolution of the total sound speed squared c_a^2 of the coupled dark energy fluid is plotted as a function of redshift. We observe that, c_a^2 takes negative values in the highly non-relativistic regime of the neutrinos.

c) The density contrast in neutrinos δ_ν, and cold dark matter δ_{CDM} is plotted on a scale of $k = 10^{-3}\ \mathrm{Mpc}^{-1}$ for $n = 0.1$ and $n = 0.01$, respectively. For both cases the neutrino mass variation is most severe at late times leading to a large coupling at late times. In the case of $n = 0.1$ the coupling is so large at present that instabilities have effectively set in (cf. the discussion in the log-linear model), whereas in the case of $n = 0.01$ the model can be regarded as stable until the present time.
It is found that the density contrast in MaVaNs grows just as in uncoupled neutrinos in General Relativity as long as the coupling remains moderate. The reason is that the effects of the scalar field on the neutrino perturbations are subdominant with respect to the gravitational influence of CDM and baryons. As a result, the growth of δ_ν with time remains moderate and δ_ν turns out to be of comparable size as δ_{CDM} today. It has to be noted that we are looking at large scales on which perturbations are suppressed by the large value of k^{-1}. However, compared to a steeply growing coupling this effect is small as can also be seen towards the present for the case of $n = 0.1$ where the mass variation is large.
In addition, according to our analytical calculation in sec. 3 the stability condition is fulfilled on scales were possible instabilities grow fastest. As argued in sec. 4.1, the CDM perturbations are known to be linear at the scale considered and thus the neutrino perturbations can also be viewed as linear until the present time in the case of n=0.01.

In conclusion, fig. 6 and fig. 7 demonstrate that the adiabatic power-law model is characterised by a growing mass at late times. For moderate coupling strengths, the

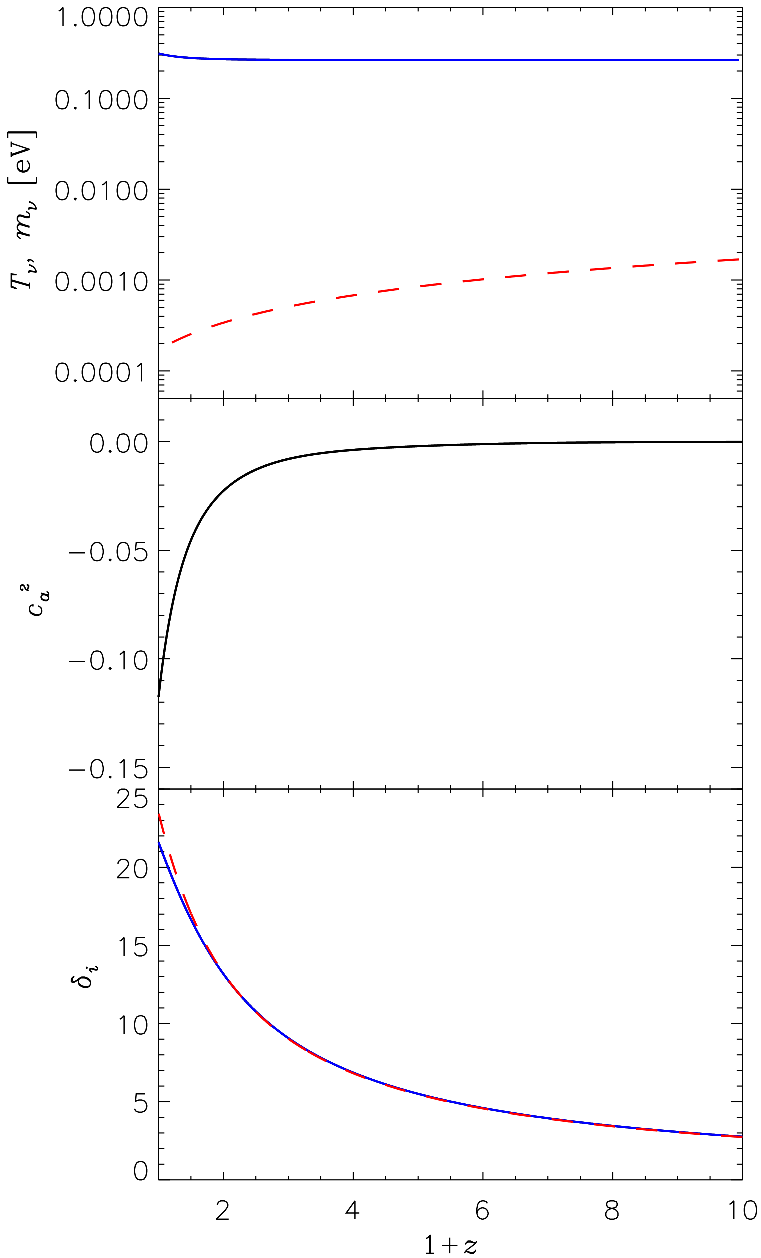

Figure 6. *a*) Neutrino mass m_ν (solid) and temperature T_ν (dotted) as a function of redshift. *b*) Total dark energy sound speed squared c_a^2 as a function of redshift. *c*) Density contrast in neutrinos (dashed) δ_ν, and density contrast in CDM $\delta_{\rm CDM}$ (solid) as a function of redshift on a scale $k = 10^{-3}\,{\rm Mpc}^{-1}$. We have used $n = 0.01$ and $M = 0.0021$ eV.

neutrino density contrast follows the cold dark matter density contrast and the model can be regarded as stable - even in the case of imaginary sound speed. However, it depends on the choice of the model parameters whether the model will remain stable in the future.

These results extends the analytical considerations of Ref. [63]. The nonlinear collapse does not happen as soon as the neutrinos become non-relativistic, as baryons and especially CDM, are able to attract the neutrinos in their potential wells formed through conventional gravitational collapse. It is important to consider the magnitude and the growth rate of

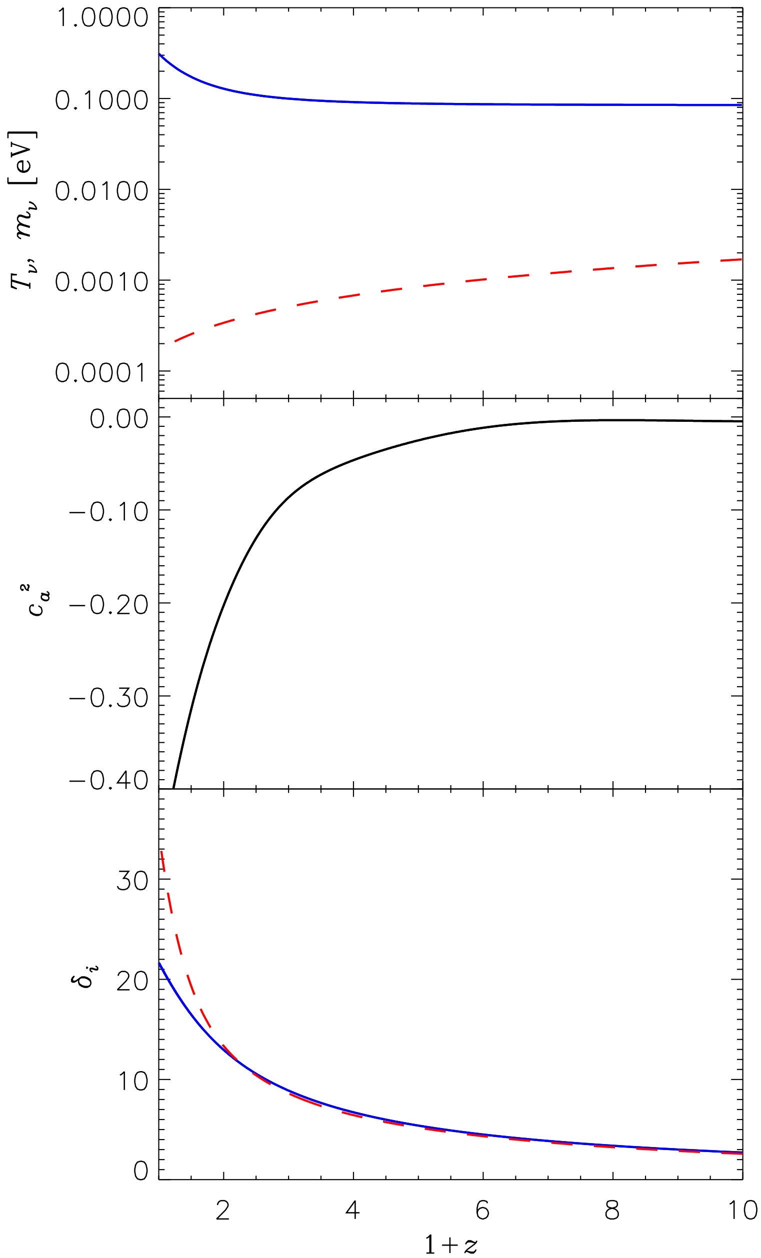

Figure 7. *a*) Neutrino mass m_ν (solid) and temperature T_ν (dotted) as a function of redshift. *b*) Total dark energy sound speed squared c_a^2 as a function of redshift. *c*) Density contrast in neutrinos (dashed) δ_ν, and density contrast in CDM $\delta_{\rm CDM}$ (solid) as a function of redshift on a scale $k = 10^{-3}\,{\rm Mpc}^{-1}$. We have used $n = 0.1$ and $M = 0.011$ eV.

the scalar field-neutrino coupling and to compare its importance relative to other sources of gravitational attraction. As indicated in the previous section, the comparison can be made quantitatively through Eq. (26).

This conclusion is further underlined by fig.8 in which we can see the cold dark matter term $\Omega_{\rm CDM}\delta_{\rm CDM}$ from Eq. (26) dominating over the coupling term $\Omega_\nu \frac{G_{\rm eff}}{G}\delta_\nu$ at early times. In this regime the model is stable. At later times the coupling term overtakes the cdm term as the coupling increases. This effect clearly makes the $n = 0.1$ model unstable.

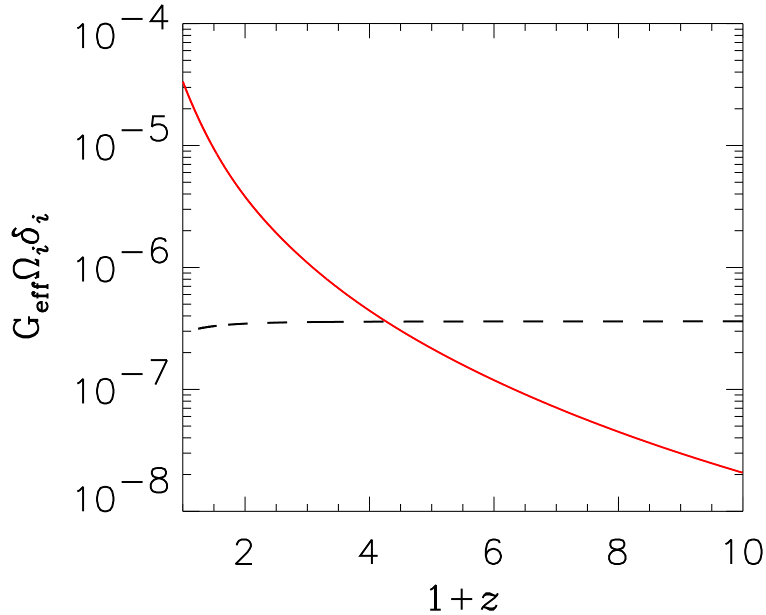

Figure 8. Comparison of the terms from Eq. (26) $\Omega_\nu \frac{G_{\text{eff}}}{G}\delta_\nu$(solid) and $\Omega_{\text{CDM}}\delta_{\text{CDM}}$(dashed) as a function of redshift on a scale $k = 10^{-3}\,\text{Mpc}^{-1}$. For $n = 0.1$ the coupling term is larger than the cdm term from a redshift of $z + 1 \sim 4$. We have used $M = 0.0021\frac{M_{\text{pl}}^{-1/2}}{\text{Mpc}^{-1/2}}$ and $\sigma = 100M_{\text{pl}}^{-2}$.

It should be noted that in both cases for the power-law model, the scalar field mass is decreasing such that in the very near future $m_\phi < H$. This means that the scale on which the perturbations are adiabatic will quickly be pushed outside the horizon and the perturbations become non-adiabetic on all scales - for a discussion of the stability of non-adiabatic models see [86].

4.3. A no-go theorem for mass varying neutrinos?

In the following, we will comment on a no-go theorem in Ref. [63] which states that any realistic adiabatic MaVaN model with $m_\phi^2 > 0$ cannot be stable for non-relativistic neutrinos.

For its deduction the authors of Ref. [63] proceeded in the following way. They derived an expression for the total sound speed squared c_a^2 in the kinetic theory picture for $p_\nu \ll m_\nu$ assuming the perturbations to be plane waves. Independent of the choice of the scalar-neutrino coupling and the scalar potential which characterise a MaVaN model, c_a^2 turned out to be negative. No reference was made of the relative gravitational importance of other relevant cosmic components like CDM and baryons.

In the present work we have shown examples of models which demonstrate that a detailed analysis of the potential and coupling functions as well as an assessment of the influence of CDM and baryons, are necessary in order to predict the growth of structure in neutrinos. In sec. 3 we found that the density contrast in neutrinos in the small scale limit only grows exponentially if the scalar-neutrino coupling is larger than all other relevant

parameters, leading to negligible growth-slowing effects as provided by cosmic expansion and CDM gravitational drag.

In this case we verified numerically for the log-linear model of the last section that c_a^2 turns negative in agreement with the result of [63]. We would like to point out that finite temperature effects which can play a crucial role for the stability of a MaVaN model [71] were included in our calculation.

However, as demonstrated by the result for the power-model, for a moderate coupling, the evolution of the neutrino density contrast is very similar to the uncoupled case in ordinary General Relativity. This was shown even in the case of an imaginary sound speed of the dark energy fluid. Accordingly, the perturbations were found to grow much slower than exponentially with time.

We would furthermore like to point out that our numerical analysis was very conservative in the sense that it assumes a comparatively large neutrino mass scale of $\sum m_\nu \sim 1$ eV. Thus, we would like to stress that the stabilising effects exerted by other cosmic components on the MaVaN perturbations can be much more efficient, in case the absolute neutrino mass scale realized in nature turns out to be in the sub-eV range.

Based on our analysis we conclude that viable adiabatic MaVaN models can be found which are stable until the present time. We indicate $\Omega_\nu \ll \Omega_{\rm CDM}$ as the main cause, since it enhances the stabilising influence exerted by CDM on the neutrino density contrast. Consequently, the dynamics in stable models are governed by CDM, largely independent of the sign of the sound speed squared, even in the highly non-relativistic regime.

Furthermore, we have integrated the relevant equations using CMBFAST and CAMB which work in the linear regime. Consequently, the mass of the scalar field had to be chosen small enough (however $\gg H$) to push the scales where possible instabilities could occur into the linear regime.

By increasing the scalar field mass and thus reducing the range of the scalar field, we would expect a local scalar field induced enhancement of the gravitational clustering of neutrinos in the non-linear regime (on scales where neutrino free-streaming cannot inhibit the growth of perturbations). Accordingly, resulting neutrino bound states would be interpreted as a contribution to the CDM small scale structure, which however, on average does not affect the equation of state of neutrino dark energy. Similarly, in chameleon cosmologies such an enhanced small scale growth of the CDM density contrast is predicted [85] due to the coupling to a scalar field with range $a/k = 250$ pc today. We thus refer to another interesting class of possibly stable MaVaN models characterised by a much larger scalar field mass. However, the detailed discussion of these models and their phenomenological implications lies beyond the scope of this paper.

5. Discussion

Models of neutrinos coupled to a light scalar field have been invoked to naturally explain the observed cosmic acceleration as well as the origin of dynamical neutrino masses. However, the class of MaVaN models characterised by an adiabatic evolution of perturbations in the non-relativistic regime may suffer from instabilities and as a result cease to act as dark

energy. In this paper we analysed the stability issue in the framework of linear perturbation theory.

For this purpose we derived the equation of motion of the density contrast in the non-relativistic neutrino regime in terms of the characteristic MaVaN functions, namely the scalar potential, the scalar-neutrino coupling, and the source terms provided by CDM and baryons. Furthermore, we modified both the CMBFAST [82] and CAMB [83] code to include a light scalar field coupled to neutrinos and numerically focused on two significant MaVaN models.

We found that the instabilities in the neutrino density contrast only occur if the influence of the scalar-neutrino coupling on the dynamics of the perturbations dominates over the growth-slowing effects (dragging) provided by CDM. As long as the coupling is moderate, the neutrinos feel a gravitational drag towards the potential wells formed by CDM. This effect can postpone the instabilities and stabilise the perturbations until today as long as the coupling remains of a moderate size. This result is largely independent of the sign of the sound speed squared.

These results were obtained from considering representative limiting cases for the time dependence of the coupling. At first, we investigated MaVaN models characterised by a strong growth of the coupling and thus of the neutrino masses with time. In this case, at late times any growth-slowing effects on the perturbations provided by the cosmic expansion or the gravitational drag of CDM can be neglected. Consequently, independent of the choice of the scalar potential, the analytic equation for the evolution of the neutrino density contrast at late times involved a faster than exponentially growing solution. Our numerical results for such a model with logarithmic scalar potential illustrated that the onset of the instability is around the time when the neutrinos turn non-relativistic. In this case, the instability could be seen as the effect of the adiabatic sound speed squared becoming negative.

Since the attraction between neutrinos increases rapidly, the sound speed changes sign as soon as the counterbalancing pressure forces in neutrinos have dropped sufficiently. As a result, the non-relativistic neutrino density contrast is inevitably driven into the non-linear regime with the likely outcome of the formation of neutrino nuggets [63].

However, we demonstrated analytically that this result does not hold true if the scalar-neutrino coupling in a MaVaN model is not strong enough to overcompensate for the growth-slowing effects provided by other cosmic components. More precisely, the stability condition was found to translate into an upper bound on the scalar neutrino coupling which is determined by the ratio between the dark matter plus baryon density to the neutrino density.

Accordingly, the value of the allowed coupling strength depends on the absolute neutrino mass scale realized in nature, its maximal value being $\beta/M_{\rm pl} \sim 0(100)$ for a minimal hierarchical neutrino mass spectrum. In this case, even though the field lies adiabatically in the minimum of the effective potential, the evolution of the neutrino density contrast at late times and up to the present epoch tends to follow the CDM density contrast just as in the uncoupled case.

Specifically, we demonstrated numerically that for the choice of a moderately growing

coupling and an inverse power law scalar potential up to the present time the neutrino density contrast is still in the linear regime on scales where possible instabilities would grow fastest. Accordingly, we have identified an adiabatic MaVaN model which can be viewed as stable until the present time.

Acknowledgments

LS thanks Andreas Ringwald for advice, continuous support and discussions and Christof Wetterich and Yong-Yeon Keum for fruitful discussions. Furthermore, we acknowledge the use of the publicly available CMBFAST and CAMB packages [82, 83]. DFM acknowledges support from the Alexander von Humboldt Foundation and from the Research Council of Norway through project number 159637/V30.

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